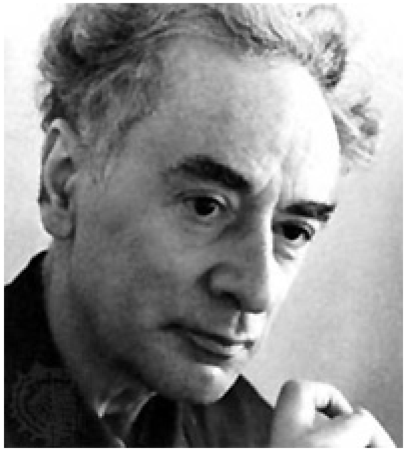
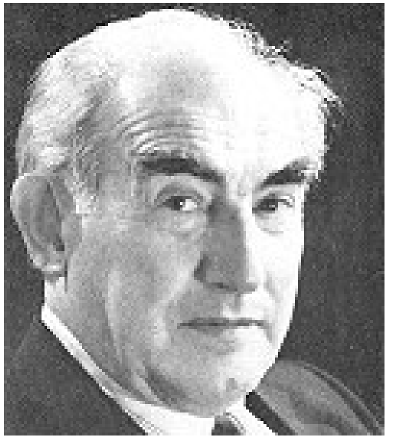
Transiciones de Fase:
Teoría de Ginzburg-Landau de Superconductividad.

- Repaso Fenomenológico de la Superconductividad.
- Teoría de Ginzburg-Landau.
- Sistemas no homogéneos: Ecuaciones de GL.
- Aplicaciones de la Ecuaciones de GL: Longitud de coherencia.
- Otras consecuencias: Cuantización del flujo.
Efecto Josephson.



Premio Nobel 1962



Premio Nobel 2003

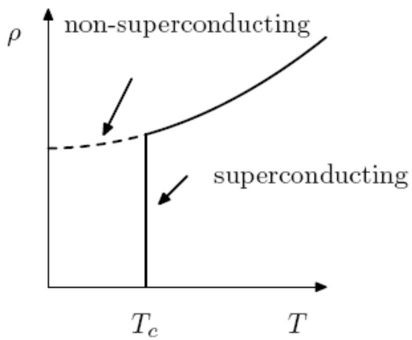


Fig. 1.1 Resistivity of a typical metal as a function of temperature. If it is a non-superconducting metal (such as copper or gold) the resistivity approaches a finite value at zero temperature, while for a superconductor (such as lead, or mercury) all signs of resistance disappear suddenly below a certain temperature, T_c.

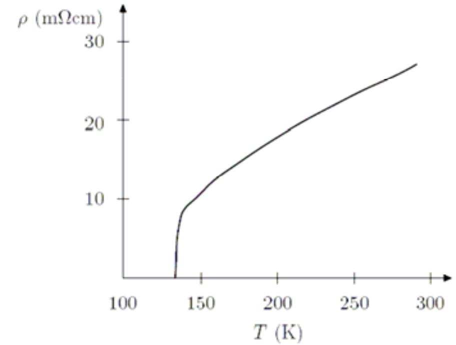


Fig. 1.2 Resistivity of $HgBa_2Ca_2Cu_3O_8 + \delta$ as a function of temperature (adapted from data of Chu (1993). Zero resistance is obtained at about 135K, the highest known T_c in any material at normal pressure. In this material T_c approaches a maximum of about 165K under high pressure. Note the rounding of the resistivity curve just above T_c, which is due to superconducting fluctuation effects. Also, well above T_c the resistivity does not follow the expected Fermi liquid behaviour.

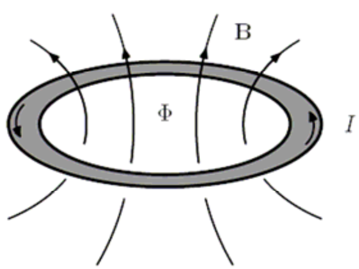


Fig. 1.4 Persistent current around a superconducting ring. The current maintains a constant magnetic flux, Φ, through the superconducting ring.

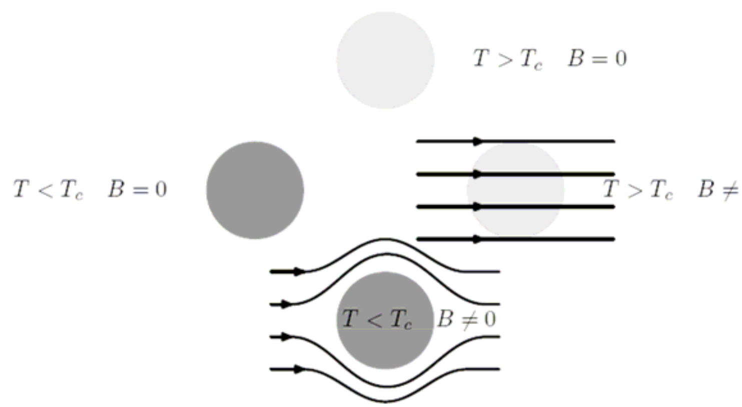


Fig. 1.5 The Meissner-Ochsenfeld effect in superconductors. If a sample initially at high temperature and in zero magnetic field (top) is first cooled (left) and then placed in a magnetic field (bottom), then the magnetic field cannot enter the material (bottom). This is a consequence of zero resistivity. On the other hand a normal sample (top) can be first placed in a magnetic field (right) and **then** cooled (bottom). In the case the magnetic field is **expelled** from the system.

substance	Tc (K)	
Al	1.2	
Hg	4.1	first superconductor, discovered 1911
Nb	9.3	higest T_c of an element at normal pressure
Pb	7.2	
Sn	3.7	
Ti	0.39	
Tl	2.4	
V	5.3	
W	0.01	
Zn	0.88	
Zr	0.65	
Fe	2	high pressure
H	300	predicted, under high pressure
O	30	high pressure, maximum T_c of any element
S	10	high pressure
Nb_3Ge	23	A15 structure, highest known T_c before 1986
$Ba_{1-x}Pb_xBiO_3$	12	first perovskite oxide structure
$La_{2-x}Sr_xCuO_4$	35	first high T_c superconductor
$YBa_2Cu_3O_{7-\delta}$	92	first superconductor above 77K
$HgBa_2Ca_2Cu_3O_{8+\delta}$	135-165	highest T_c ever recorded
$K_3\ C_{60}$	30	fullerene molecules
YNi_2B_2C	17	borocarbide superconductor
MgB_2	38	discovery announced in January 2001
Sr_2RuO_4	1.5	possible p-wave superconductor
UPt_3	0.5	"heavy fermion" exotic superconductor
$(TMTSF)_2ClO_4$	1.2	organic molecular superconductor
ET-BEDT	12	organic molecular superconductor

Efecto Meisner: Ecuación de London.

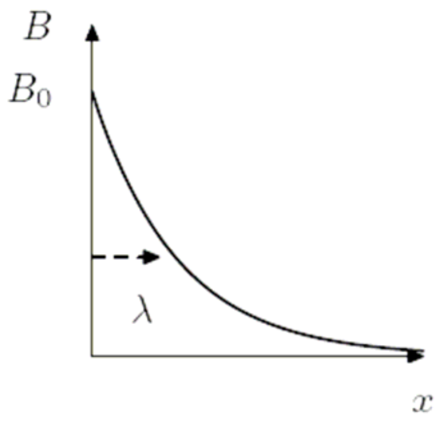
$$\mathbf{j} = -\frac{n_s e^2}{m_e}\mathbf{A}.$$

$$\nabla \times \mathbf{j} = -\frac{n_s e^2}{m_e}\mathbf{B}.$$

$$\nabla \times \mathbf{B} = \mu_o \mathbf{j}.$$

$$\nabla \times (\nabla \times \mathbf{B}) = -\mu_o \frac{n_s e^2}{m_e}\mathbf{B}$$

$$\nabla \times (\nabla \times \mathbf{B}) = -\frac{1}{\lambda^2}\mathbf{B}$$



$$B = B_0 \exp\left(-x/\lambda\right)$$

Efecto Meisner: Ecuación de Pippard.

$$\mathbf{j}(\mathbf{r}) = -\frac{n_s e^2}{m_e} \frac{3}{4\pi\xi_0} \int \frac{\mathbf{R}(\mathbf{R}.\mathbf{A}(\mathbf{r}'))}{R^4} e^{-R/r_0} d^3 r',$$

$$\frac{1}{r_0} = \frac{1}{\xi_0} + \frac{1}{l}.$$

Longitud de coherencia.

$$l = v_F \tau,$$

$$\kappa = \lambda/\xi_0$$

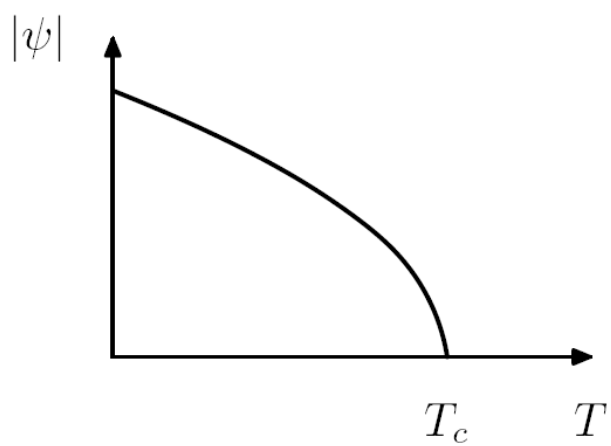
Teoría de Ginzburg-Landau.

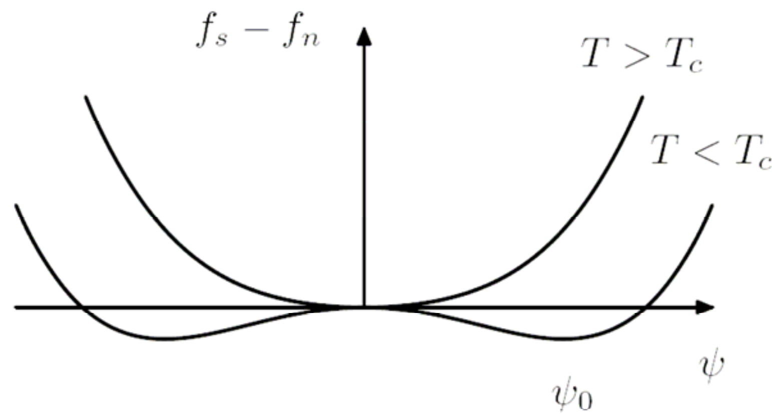
El parámetro de orden es una cantidad compleja

$$\psi = \begin{cases} 0 & T > T_c \\ \psi(T) \neq 0 & T < T_c. \end{cases}$$

$$f_s(T) = f_n(T) + a(T)|\psi|^2 + \frac{1}{2}b(T)|\psi|^4 + \dots$$

$$\begin{aligned} a(T) &\approx \dot{a} \times (T - T_c) + \dots \\ b(T) &\approx b + \dots, \end{aligned}$$





Generalización a sistemas inhomogéneos:

$$f_s(T) = f_n(T) + \frac{\hbar^2}{2m^*}|\nabla\psi(\mathbf{r})|^2 + a(T)|\psi(\mathbf{r})|^2 + \frac{b(T)}{2}|\psi(\mathbf{r})|^4$$

$$F_s(T) = F_n(T) + \int d^3r \left(\frac{\hbar^2}{2m^*}|\nabla\psi|^2 + a(T)|\psi(\mathbf{r})|^2 + \frac{b(T)}{2}|\psi(\mathbf{r})|^4\right) d^3r.$$

$$\psi(\mathbf{r}) \rightarrow \psi(\mathbf{r}) + \delta\psi(\mathbf{r})$$

$$\begin{aligned} \delta F_s \quad = \quad & \int \left[\frac{\hbar^2}{2m^*}(\nabla\delta\psi^*).(\nabla\psi) + \delta\psi^*(a\psi + b\psi|\psi^2|)\right] d^3r \\ & + \int \left[\frac{\hbar^2}{2m^*}(\nabla\psi^*).(\nabla\delta\psi) + (a\psi^* + b\psi^*|\psi^2|)\delta\psi\right] d^3r. \end{aligned}$$

Integrando por partes y (usando el teorema de Gauss):

$$\delta F_s = \int \delta\psi^* \left(-\frac{\hbar^2}{2m^*} \nabla^2 \psi + a\psi + b\psi|\psi^2| \right) d^3r$$
$$+ \int \left(-\frac{\hbar^2}{2m^*} \nabla^2 \psi + a\psi + b\psi|\psi^2| \right)^* \delta\psi d^3r.$$

$$\delta F = 0 \Big| \qquad -\frac{\hbar^2}{2m^*} \nabla^2 \psi + a\psi + b\psi|\psi^2| = 0.$$

Ecuación de Schrödinger no lineal.

En la integración por partes:

$$\vec{\nabla}(\delta\psi^*)\vec{\nabla}\psi = \vec{\nabla}(\delta\psi^*\vec{\nabla}\psi) - \delta\psi^*\nabla^2\psi$$

Derivada funcional:

$$F[\phi(\mathbf{x})] = \int d\mathbf{x} f(\phi(\mathbf{x}), \nabla\phi(\mathbf{x})),$$

$$\begin{aligned}
\frac{\delta F}{\delta\phi(\mathbf{y})} &= \int d^d x \frac{\delta f}{\delta\phi(\mathbf{y})} \\
&= \int d^d x \left[\frac{\partial f}{\partial\phi(\mathbf{x})} \frac{\delta\phi(\mathbf{x})}{\delta\phi(\mathbf{y})} + \frac{\partial f}{\partial\nabla\phi(\mathbf{x})} \cdot \frac{\delta\nabla\phi(\mathbf{x})}{\delta\phi(\mathbf{y})} \right] \\
&= \int d^d x \left[\frac{\partial f}{\partial\phi(\mathbf{x})} \delta(\mathbf{x}-\mathbf{y}) + \frac{\partial f}{\partial\nabla\phi(\mathbf{x})} \cdot \nabla\delta(\mathbf{x}-\mathbf{y}) \right].
\end{aligned}$$

$$\frac{\delta F}{\delta\phi(\mathbf{y})} = \frac{\partial f}{\partial\phi(\mathbf{y})} - \nabla \cdot \frac{\partial f}{\partial\nabla\phi(\mathbf{y})}$$

Superficie de separación normal-superconductor.

$$-\frac{\hbar^2}{2m^*}\frac{d^2\psi(x)}{dx^2} + a(T)\psi(x) + b(T)\psi^3(x) = 0 \qquad (2.40)$$

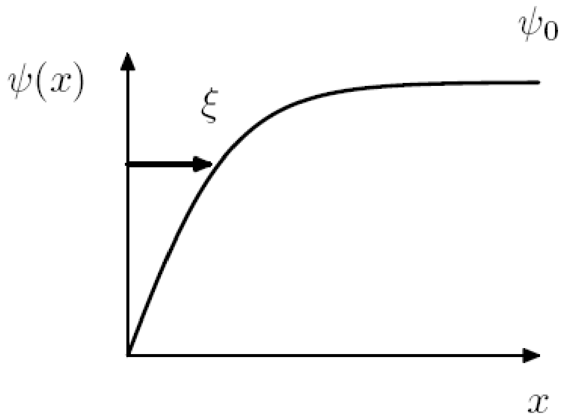
in the region $x > 0$ with the boundary condition at $\psi(0) = 0$. It turns out that one can solve this equation directly (exercise 4.2) to find

$$\psi(x) = \psi_0 \tanh\left(\frac{x}{\sqrt{2}\xi(T)}\right), \qquad (2.41)$$

$$\xi(T) = \left(\frac{\hbar^2}{2m^*|a(T)|}\right)^{1/2}.$$

Longitud de coherencia de GL

$$\xi(T) = \xi(0)|t|^{-1/2},$$



Teoría GL con campo magnético: $\frac{\hbar}{i}\boldsymbol{\nabla} \rightarrow \frac{\hbar}{i}\boldsymbol{\nabla} - q\mathbf{A}$

$$f_s(T) = f_n(T) + \frac{\hbar^2}{2m^*}|(\frac{\hbar}{i}\boldsymbol{\nabla} + 2e\mathbf{A})\psi|^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4.$$

$$F_s(T) = F_n(T) + \int \left(\frac{\hbar^2}{2m^*}|(\frac{\hbar}{i}\boldsymbol{\nabla} + 2e\mathbf{A})\psi|^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4\right) d^3r + \frac{1}{2\mu_0}\int B(\mathbf{r})^2 d^3r. \quad (2.49)$$

La derivada funcional respecto de ψ nos da la NLSE.

$$-\frac{\hbar^2}{2m^*}(\nabla + \frac{2ei}{\hbar}\mathbf{A})^2\psi(\mathbf{r}) + \left(a + b|\psi|^2\right)\psi(\mathbf{r}) = 0.$$

Y la derivada funcional respecto del potencial vector nos da:

$$\mathbf{j}_s = -\frac{\partial F_s}{\partial \mathbf{A}(\mathbf{r})} \Bigg|$$

$$\mathbf{j}_s = -\frac{2e\hbar i}{2m^*}\left(\psi^* \boldsymbol{\nabla}\psi - \psi \boldsymbol{\nabla}\psi^*\right) - \frac{(2e)^2}{m^*}|\psi|^2 \mathbf{A}.$$

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0(\mathbf{j}_{ext} + \mathbf{j}_s),$$

Invarianza Gauge: consideremos el parámetro de orden complejo

$$\psi(\mathbf{r}) = |\psi(\mathbf{r})| e^{i\theta(\mathbf{r})}.$$

Si realizamos una transformación gauge del potencial vector:

$$\mathbf{A}(\mathbf{r}) \to \mathbf{A}(\mathbf{r}) + \boldsymbol{\nabla}\chi(\mathbf{r})$$

Consideremos el operador momento: $$\hat{p} = \frac{\hbar}{i}\boldsymbol{\nabla} + 2e\mathbf{A}.$$

Si cambiamos el parámetro de orden: $$\psi(\mathbf{r}) \to \psi(\mathbf{r}) e^{i\theta(\mathbf{r})}$$

$$\begin{aligned}
\hat{p}\psi(\mathbf{r})e^{i\theta(\mathbf{r})} &= e^{i\theta(\mathbf{r})}\left(\frac{\hbar}{i}\boldsymbol{\nabla} + 2e\mathbf{A}\right)\psi(\mathbf{r}) + \psi(\mathbf{r})e^{i\theta(\mathbf{r})}\hbar\boldsymbol{\nabla}\theta(\mathbf{r}) \\
&= e^{i\theta(\mathbf{r})}\left(\frac{\hbar}{i}\boldsymbol{\nabla} + 2e(\mathbf{A} + \frac{\hbar}{2e}\boldsymbol{\nabla}\theta)\right)\psi(\mathbf{r}).
\end{aligned}$$

El cambio en potencial vector para que todo quede invariante

$$\mathbf{A}(\mathbf{r}) \to \mathbf{A}(\mathbf{r}) + \frac{\hbar}{2e}\boldsymbol{\nabla}\theta.$$

Si tomamos que las única variaciones de ψ corresponden a la fase:

$$F_s = F_s^0 + \rho_s \int d^3r \left(\boldsymbol{\nabla}\theta + \frac{2e}{\hbar}\mathbf{A}\right)^2.$$

$$\rho_s = \frac{\hbar^2}{2m^*}|\psi|^2$$

De aquí

$$\begin{aligned} \mathbf{j}_s &= -\frac{\partial F_s[\mathbf{A}]}{\partial \mathbf{A}(\mathbf{r})} \\ &= -\frac{2e}{\hbar}\rho_s\left(\nabla\theta + \frac{2e}{\hbar}\mathbf{A}\right). \end{aligned}$$

Si θ = cte en todo el superconductor

$$\mathbf{j}_s = -\rho_s \frac{(2e)^2}{\hbar^2}\mathbf{A}$$

Esto es la ecuación de London !!!

$$\mathbf{j}_s = -\frac{(2e)^2}{2m^*}|\psi|^2\mathbf{A},$$

$$\mathbf{j}_s = -\frac{n_s e^2}{m_e}\mathbf{A}.$$

Comparando: $n_s = 2|\psi|^2$ $m^* = 2m_e$

$$n_s = 2|\psi^2| = 2\frac{\dot{a}(T_c - T)}{b}.$$

$$\lambda(T) = \left(\frac{m_e b}{2\mu_0 e^2 \dot{a}(T_c - T)}\right)^{1/2}.$$

$$\kappa = \frac{\lambda(T)}{\xi(T)},$$

Table 2.1 Penetration depth, $\lambda(0)$, and coherence length, $\xi(0)$, at zero temperature for some important superconductors. Data values are taken from Poole (2000).

	T_c (K)	[redacted](0) (nm)	[redacted](0)(nm)	κ
Al	1.18	1550	45	0.03
Sn	3.72	180	42	0.23
Pb	7.20	87	39	0.48
Nb	9.25	39	52	1.3
Nb_3Ge	23.2	3	90	30
YNi_2B_2C	15	8.1	103	12.7
K_3C_{60}	19.4	2.8	240	95
$YBa_2Cu_3O_{7-\delta}$	91	1.65	156	95

ξ λ

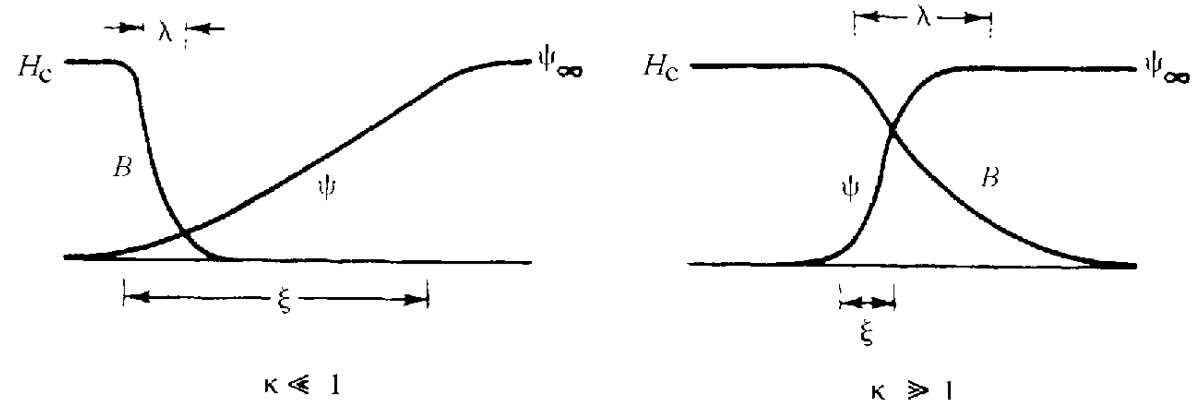
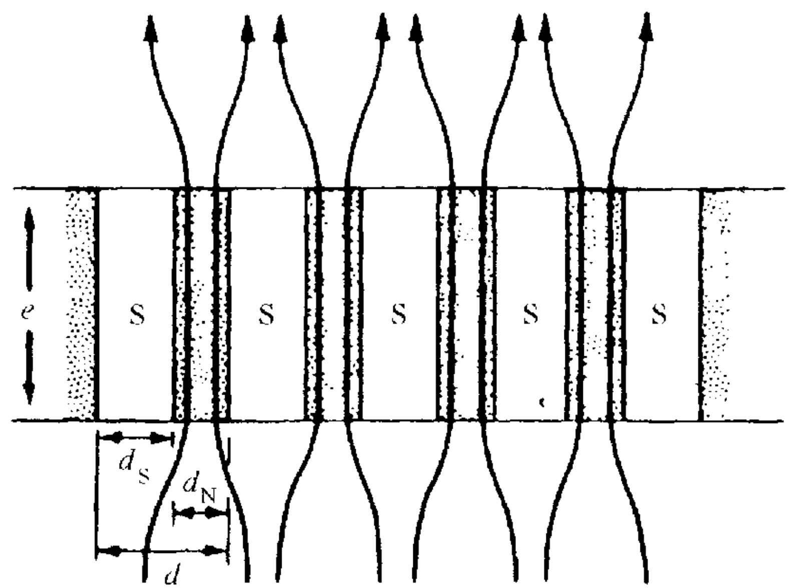


Figure 9.2 Schematic diagram of the variation of B and ψ in a domain wall. The case $\kappa << 1$ refers to a Type I superconductor with a positive surface energy; the case $\kappa >> 1$ refers to a Type II superconductor with negative surface energy.



Cuantización del flujo: Consideremos un anillo .

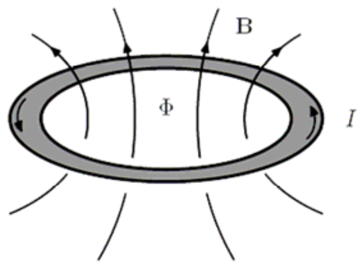


Fig. 1.4 Persistent current around a superconducting ring. The current maintains a constant magnetic flux, Φ, through the superconducting ring.

$$\Psi = |\Psi| e^{i\chi}$$

$$\mathbf{j} = -\frac{e\hbar}{m}|\Psi|^2 \left[\nabla\chi + \frac{2e}{\hbar c}\mathbf{A}\right]$$

$$\underbrace{\oint \frac{\mathbf{j}}{|\Psi|^2}\, d\mathbf{l}}_{0} = -\frac{e\hbar}{m}\left[\underbrace{\oint \nabla\chi\, d\mathbf{l}}_{-\,2\pi k} + \frac{2e}{\hbar c}\underbrace{\oint \mathbf{A}\, d\mathbf{l}}_{\text{flux}}\right]$$

$$\Phi = k\Phi_0, \quad \Phi_0 = \pi\hbar c/e = 2.07 \cdot 10^{-7}\mathrm{G} \cdot \mathrm{cm}^2.$$