Renormalización en el espacio de momentos (Huang):

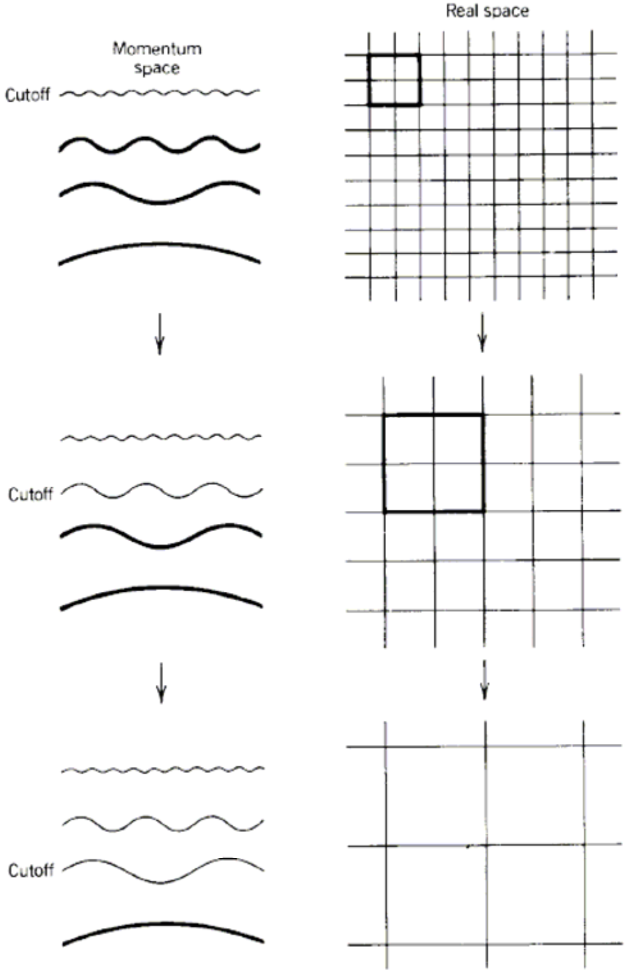


Fig. 18.5 Coarse-graining in momentum space and in real space. In the former, one effectively lowers the cutoff. In the latter, one blots out finer details, enlarging the effective lattice spacing.

$$E[m] = \int (dx)\, \psi(m(x))$$

$$\psi(m(x)) = \tfrac{1}{2} |\nabla m(x)|^2 + \sum_{n=1}^{\infty} K_n m^n(x) + \cdots$$

$$E[\tilde{m}] = \frac{1}{2} \int (dk)\left(k^2 + r_0\right) |\tilde{m}(k)|^2 + \cdots$$

$$Q \equiv e^{-G} = \mathcal{N} \prod_{|k| < \Lambda}' \int d\tilde{m}(k)\, d\tilde{m}^*(k)\, e^{-E[\tilde{m}]}$$

1. *Integration.* Define a new Hamiltonian E' by "integrating out" the k values whose magnitude lies between Λ and Λ/b $(b>1)$:

$$e^{-E'[\tilde{m}]} \equiv e^{\Omega} \prod_{\frac{\Lambda}{b}<|k|<\Lambda}' \int d\tilde{m}(k)\, d\tilde{m}^*(k)\, e^{-E[\tilde{m}]} \tag{18.55}$$

where the constant Ω is a function of Λ and all the coupling constants. The new Hamiltonian $E'[\tilde{m}]$ depends only on $\tilde{m}(k)$ with $|k|<\Lambda/b$. Apart from that, it has the same form as (18.53) with new coupling constants:

$$E'[\tilde{m}] = \frac{1}{2}\int (dk)\left(Ak^2+\tilde{r}_0\right)\left|\tilde{m}(k)\right|^2 + \cdots \tag{18.56}$$

Note that the coefficient of the k^2 term is changed. The partition function can be rewritten as

$$Q \equiv e^{-G} = e^{\Omega}\mathcal{N} \prod_{|k|<\frac{\Lambda}{b}}' \int d\tilde{m}(k)\, d\tilde{m}^*(k)\, e^{-E'[\tilde{m}]} \tag{18.57}$$

2. *Rescaling.* Restore the cutoff to Λ by increasing the unit of length by a factor b, by changing the variable of integration to

$$k' = bk \tag{18.58}$$

The Hamiltonian now reads

$$E'[\tilde{m}] = \frac{b^{-d}}{2} \int (dk') \left(\frac{A}{b^2} k'^2 + \tilde{r}_0 \right) \left| \tilde{m}\left(\frac{k'}{b} \right) \right|^2 + \cdots \tag{18.59}$$

3. *Normalization.* Restore the standard normalization of the order parameter, i.e., make the coefficient of the k'^2 term in (18.59) equal to $\frac{1}{2}$. This can be done by replacing $\tilde{m}(k)$ by

$$\tilde{m}'(k') \equiv \sqrt{\frac{A}{b^{d+2}}}\, \tilde{m}\left(\frac{k'}{b}\right) \tag{18.60}$$

which is the analog of the block-spin transformation. The final RG-transformed Hamiltonian is

$$E'[\tilde{m}'] = \frac{1}{2}\int (dk')(k'^2 + r_0')|\tilde{m}'(k')|^2 + \cdots, \qquad \left(r_0' = \frac{b^2}{A}\tilde{r}_0\right) \tag{18.61}$$

The partition function is, in terms of E',

$$Q = e^{\Omega}\mathcal{N}' \prod_{|k'|<\Lambda}' \int d\tilde{m}'(k')\, d\tilde{m}'^*(k')\, e^{-E'[\tilde{m}']} \tag{18.62}$$

where $\mathcal{N}'$ is a new normalization constant. (It is infinite both in the infinite-volume limit and the infinite-cutoff limit, but physically irrelevant.)

Hamiltoniano de Landau-Ginzburg-Wilson.

$$E[m] = \int (dx) \left\{ \tfrac{1}{2} |\nabla m(x)|^2 + \tfrac{1}{2} r_0 m^2(x) + u_0 m^4(x) \right\}$$

$$E'[\overline{m}] = \int (dx) \left[\tfrac{1}{2} |\nabla \overline{m}(x)|^2 + \tfrac{1}{2} \tilde{r}_0 \overline{m}^2(x) + \tilde{u}_0 \overline{m}^4(x) \right]$$

$$\tilde{r}_0 = r_0 + 12(\log b) C_d \left(\Lambda^{d-2} u_0 - \Lambda^{d-4} r_0 u_0 \right)$$

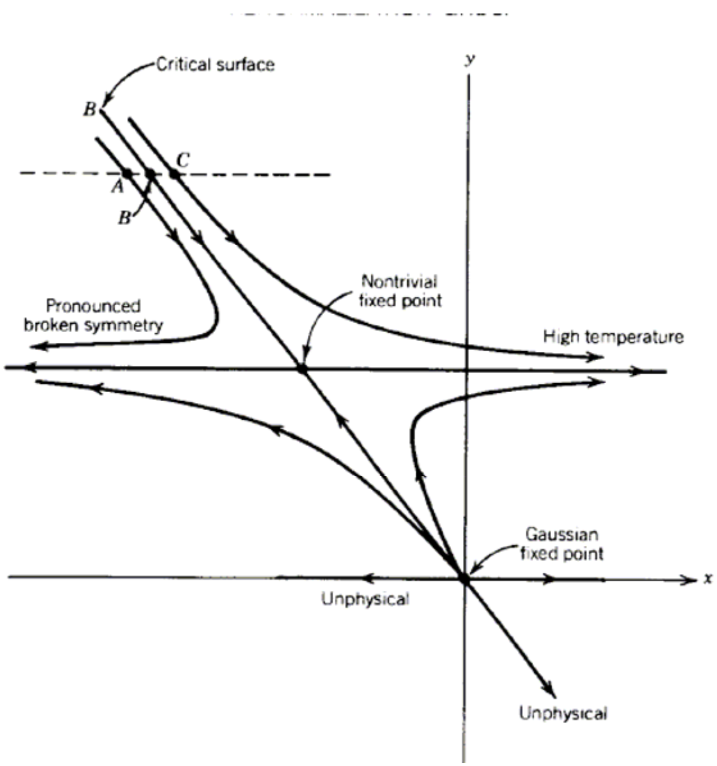
$$\tilde{u}_0 = u_0 - 36(\log b) C_d \Lambda^{d-4} u_0^2$$

Ejercicio 9.6

$$r' = 4\{r + 3cu/(1+r)\}, \tag{9.45}$$

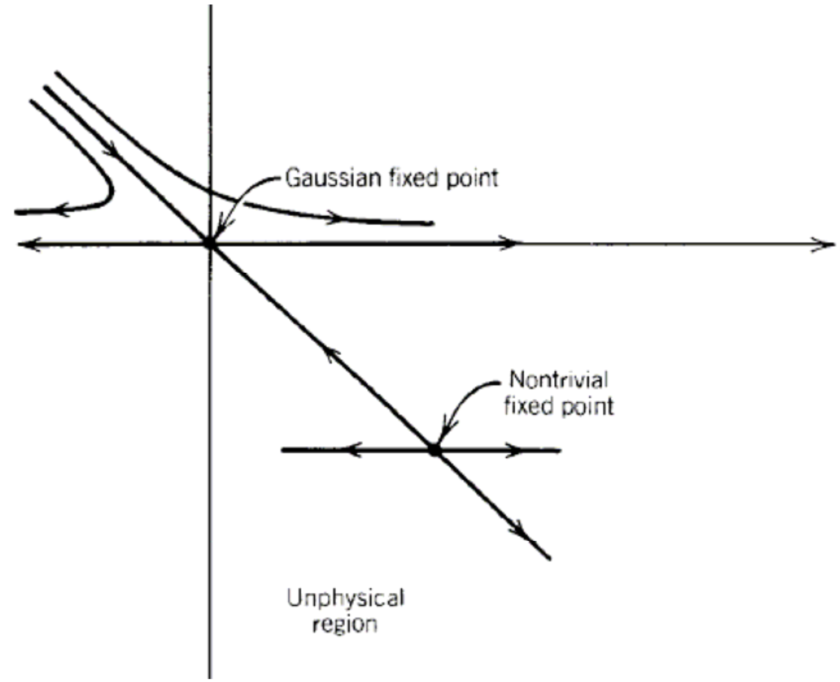
$$u' = 2^{\epsilon}\{u - 9cu^2/(1+r)^2\} \tag{9.46}$$

$d < 4$



Critical surface
y
B
C
A
B
Nontrivial fixed point
Pronounced broken symmetry
High temperature
Gaussian fixed point
x
Unphysical
Unphysical

$d > 4$



Gaussian fixed point
x
Nontrivial fixed point
Unphysical region

Modelo XY: Transición de Kosterlitz-Thouless

$$H = -J\sum_{\langle i,j\rangle} \mathbf{s}_i \cdot \mathbf{s}_j = -J\sum_{\langle i,j\rangle} \cos(\theta_i - \theta_j)$$

- Superconductividad y superfluidez en 2d.
- Ordenamiento cristalino en 2d.
- Fases en cristales líquidos en 2d.
- Melting en 2d (Modelos SOS)

$$H = -J \sum_{\langle i,j \rangle} \left(1 - \frac{(\theta_i - \theta_j)^2}{2} \right)$$

Aproximación SW

$$H = E_0 + \frac{J}{2} \int d\mathbf{r} \left(\nabla \theta (\mathbf{r}) \right)^2$$

Límite Continuo

$$Z = \int D\theta \exp \left[-\beta \frac{J}{2} \int d\mathbf{r} \left(\nabla \theta (\mathbf{r}) \right)^2 \right]$$

$$\int D\theta = \prod_n \int_{-\pi}^{\pi} d\theta_n$$

$$\theta(\mathbf{r}) = \int \frac{d\mathbf{k}}{(2\pi)^d} \hat{\theta}(\mathbf{k}) \exp^{-i\mathbf{k}\cdot\mathbf{r}}$$

$$\langle S_x \rangle = \exp \left(-\frac{T}{2Ja^{2-d}} \mathcal{S}_d \int_{\pi/L}^{\pi/a} dk k^{d-3} \right) .$$

$$I(L) = \int_{\pi/L}^{\pi/a} dk k^{d-3}. \qquad d < 2 \Big| \quad I(L) \sim L^{2-d} \to \infty \qquad \langle S_x \rangle = 0$$

$$d > 2 \Big| \quad I(L) \to A = \frac{1}{d-2} \left(\frac{\pi}{a} \right)^{d-2}$$

$$\langle S_x \rangle = \exp(-\frac{\mathcal{S}_d}{2Ja^{2-d}} AT) > 0. \Big|$$

$$d = 2 \Big| \qquad I(L) = \ln(L/a) \qquad \langle S_x \rangle = 0$$

No hay orden de largo alcance para d ≤ 2 para sistemas con simetría continua. (Teorema de Mermin-Wagner)

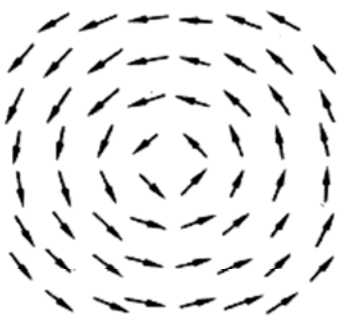
¿Como se comporta la función de correlación ?

$$\langle \mathbf{S}(\mathbf{r})\mathbf{S}(0)\rangle \simeq \begin{cases} e^{-\mathrm{const}.T} & \text{for } d>2 \\ \left(\frac{r}{L}\right)^{-\eta} & \text{for } d=2 \\ \exp(-\frac{T}{2Ja}r) & \text{for } d=1. \end{cases}$$

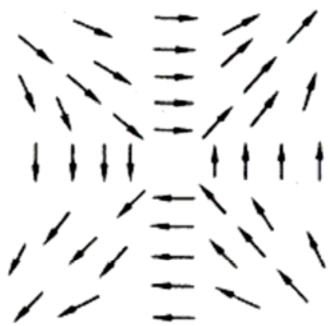
En d=2 las ondas de espín no destruyen el orden de corto alcance !!!.

Más allá de las ondas de espín. VÓRTICES.

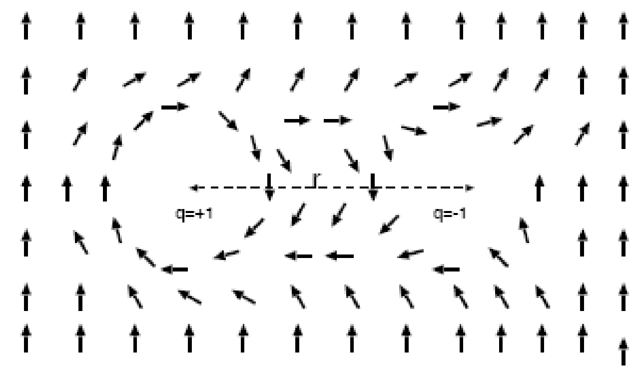
$$\oint \nabla\theta(\mathbf{r}) \cdot d\mathbf{l} = 2\pi n.$$



(a)



(b)



Energía de un vórtice.

$$2\pi n = \oint \nabla\theta(\mathbf{r}) \cdot d\mathbf{l} = 2\pi r |\nabla\theta|.$$

$$|\nabla\theta(r)| = n/r$$

$$\begin{aligned} E_{vor} - E_0 &= \frac{J}{2} \int d\mathbf{r} [\nabla\theta(\mathbf{r})]^2 \\ &= \frac{Jn^2}{2} \int_0^{2\pi} \int_a^L r dr \frac{1}{r^2} \\ &= \pi n^2 J \ln(\frac{L}{a}). \end{aligned}$$

Energía de un vórtice- antivórtice

$$E_{2vor}(R) = 2E_c + E_1 \ln(R/a).$$

Transición Kosterlitz-Thouless (KT). (o Berenzinski-KT)

$$F = E - TS. \qquad S = k_B \ln(L^2/a^2)$$

$$F = E_0 + (\pi J - 2k_B T)\ln(L/a).$$

$$T_{KT} = \pi J/2k_B$$

Por debajo de T_{KT} los vórtices no existen o están en pares de V-aV.
Por encima se separan y destruyen el orden.

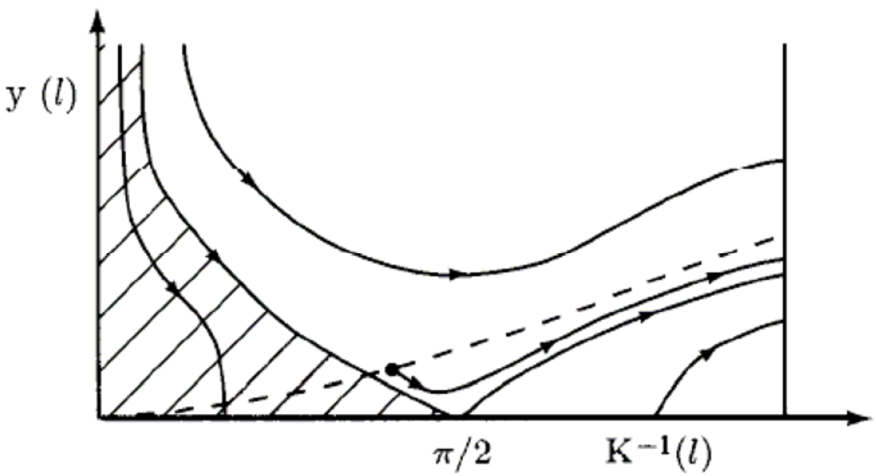
Gas de Coulomb neutro. Grupo de renormalización.

$$\frac{1}{T}H_{vor} = -\pi K \sum_{\mathbf{r}_1 \neq \mathbf{r}_2} n(\mathbf{r}_1) \ln\left(\frac{|\mathbf{r}_1 - \mathbf{r}_2|}{a}\right) n(\mathbf{r}_2) + \frac{E_c}{T} \sum_{\mathbf{r}} n(\mathbf{r})^2.$$

$$y = \exp(-\beta E_c)$$

$$\frac{d\tilde{K}^{-1}}{dl} = 2\pi^3 \tilde{y}^2$$

$$\frac{d\tilde{y}}{dl} = (2 - \pi\tilde{K})\tilde{y}.$$



$$\frac{\pi J}{k_b T_C} = 1 - 2\pi \exp\left(-\frac{\pi^2 J}{k_b T_C}\right)$$

$$\xi(T) \propto \exp(b/|T - T_c|^{\nu})$$

$\nu = ½$ (RG)

Simulaciones:

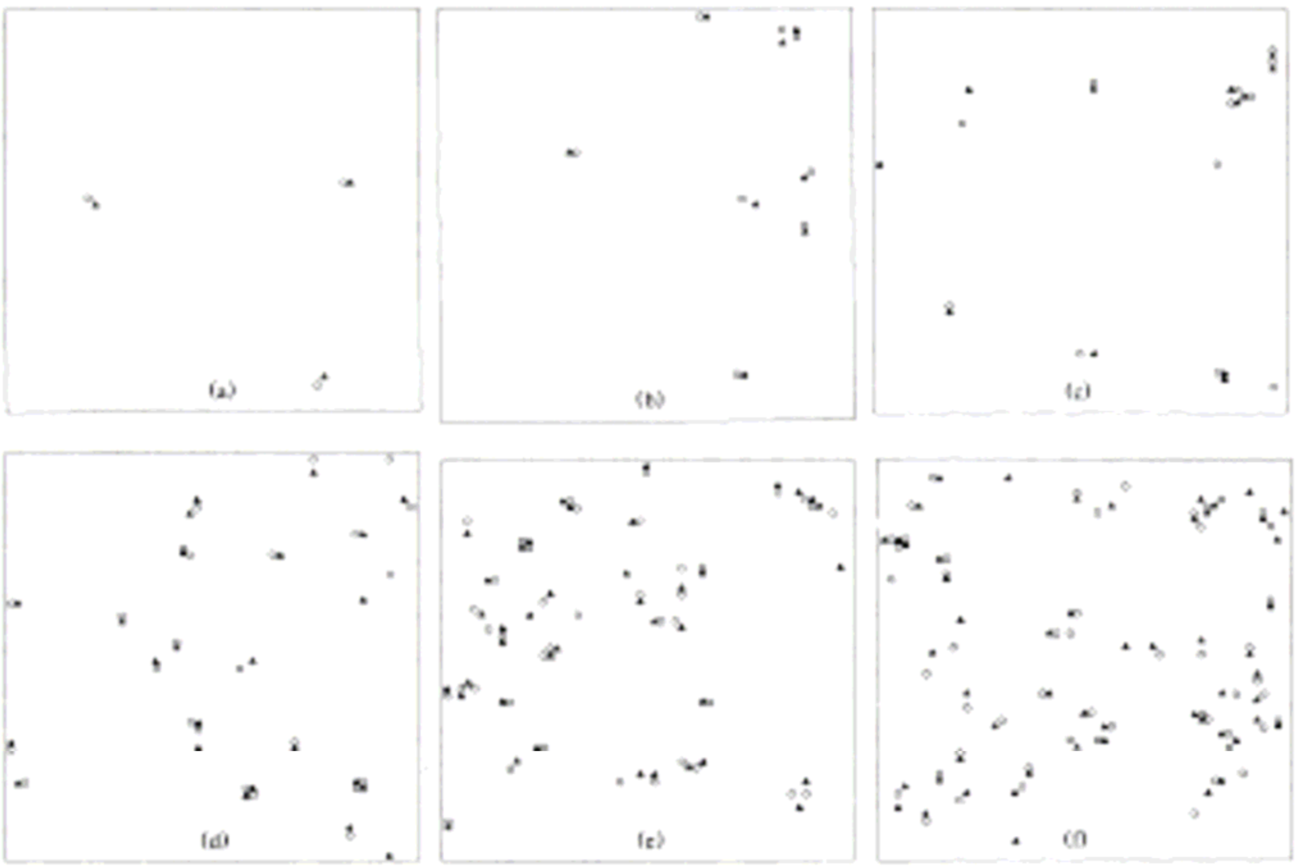
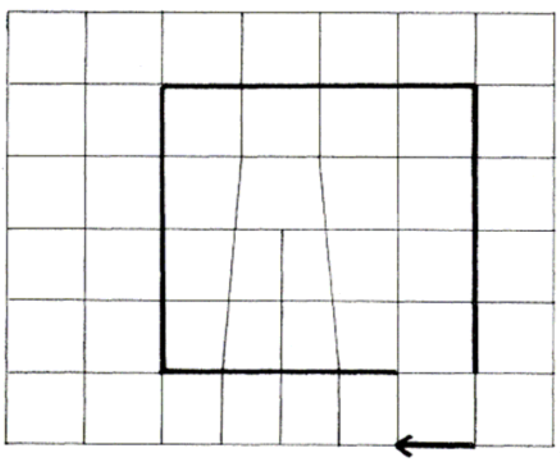


FIG. 7. Position of vortices for 3600 spin system for typical configurations. ○, positive vortices. ▲, negative vortices. (a) $T=0.80$, (b) $T=0.85$, (c) $T=0.90$, (d) $T=0.95$, (e) $T=1.00$, and (f) $T=1.05$.

“Melting” en dos dimensiones: (vórtices == dislocaciones)

“Sólido” → Hexática → Fase líquida

Dislocación.



Disclinaciones