MODELOS:

- Adoptaremos una aproximación minimalista.
- Trataremos con variables de "espín".
- Geometría en redes regulares ("lattice models").

Modelo de Ising (modelo magnético):

$$\mathrm{H} = -J\sum_{i,j} S_i S_j - H\sum_i S_i \qquad S_i = \pm 1$$

Si J = 0 .... Paramagnetismo ...

Si J > 0 ferromagnetismo
Si J < 0 antiferromagnetismo (si la red se puede dividir en subredes)

En d=1 no hay transición a T finita.
En d = 2 y d = 3 hay transición (anti) ferromagnético a paramagnético.

Aleación Cu-Zn (latón – "brass")

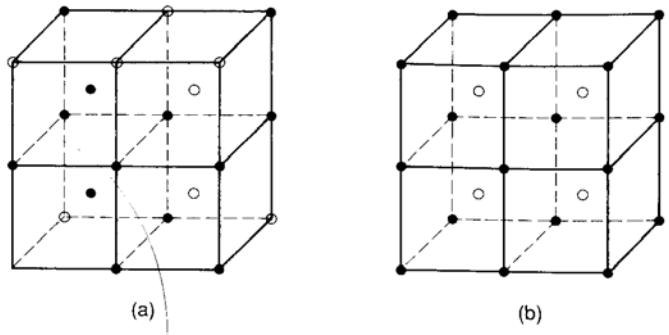


Fig. 3.2. A typical configuration of the copper and zinc atoms of beta-brass on the body-centred cubic lattice: (*a*) $T \gg T_c$; (*b*) $T \ll T_c$.

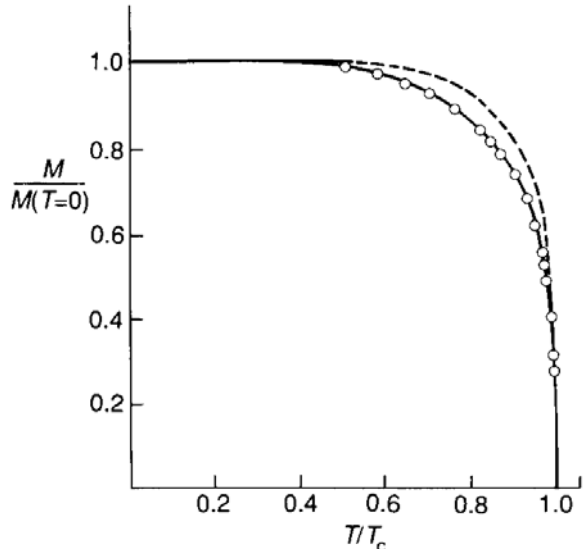


Fig. 3.3. Temperature dependence of the order parameter of beta-brass. The open circles are neutron scattering results, the dashed line X-ray scattering results, and the full line is the theoretical result for a compressible Ising model. The discrepancy between the X-ray and neutron data may arise because of the low sensitivity of X-rays to the atomic ordering. After Als-Nielsen, J. (1976). Neutron scattering and spatial correlation near the critical point. In *Phase transitions and critical phenomena*, Vol. 5a (eds C. Domb and M. S. Green), p.87. (Academic Press, London).

$s_i = 1$ if site i is occupied by a copper atom,
$s_i = -1$ if site i is occupied by a zinc atom.

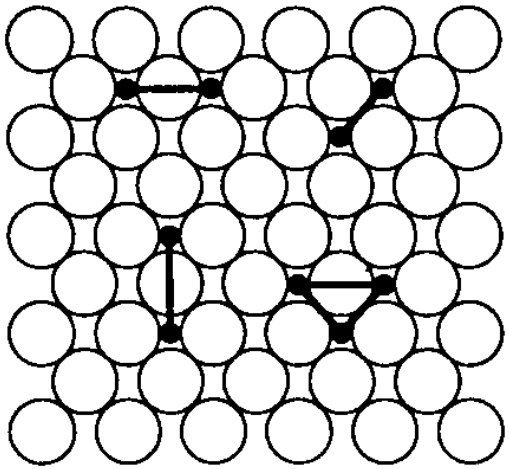
$$\mathcal{H} = \frac{1}{4}\sum_{\langle ij\rangle} J_{CuCu}(1+s_i)(1+s_j) + \frac{1}{4}\sum_{\langle ij\rangle} J_{ZnZn}(1-s_i)(1-s_j)$$
$$+ \frac{1}{4}\sum_{\langle ij\rangle} J_{CuZn}\{(1+s_i)(1-s_j) + (1-s_i)(1+s_j)\}. \qquad (3.2)$$

$$\mathcal{H} = -J\sum_{\langle ij\rangle} s_i s_j - H\sum_i s_i + C$$

Lattice gas

$$\mathcal{H} = -J_L \sum_{\langle ij \rangle} t_i t_j - \mu_L \sum_i t_i$$

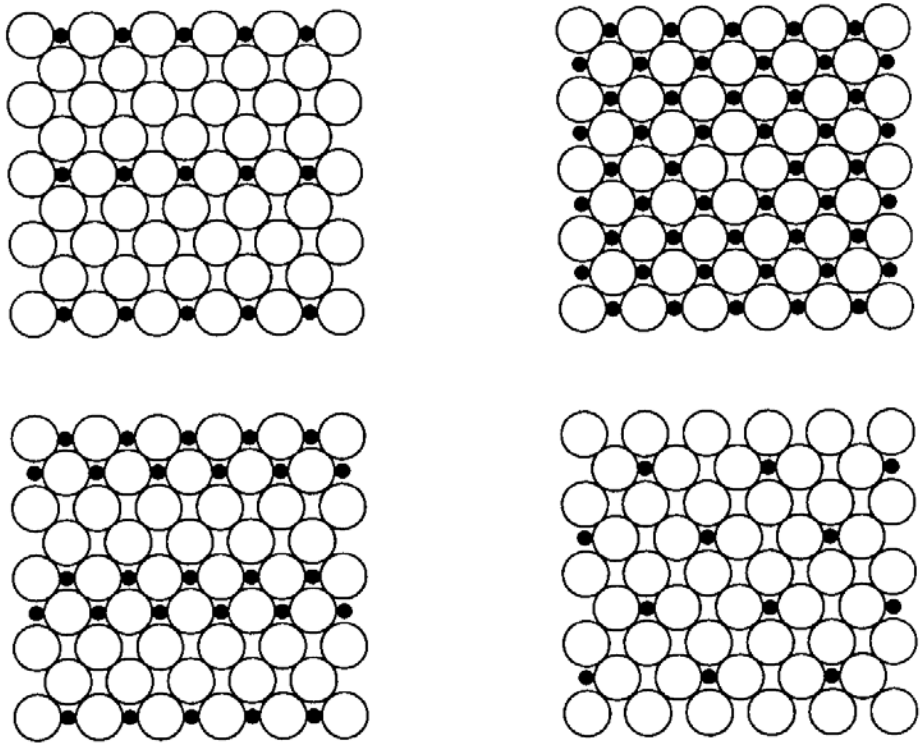
(a)



$$t_i = 1, 0$$

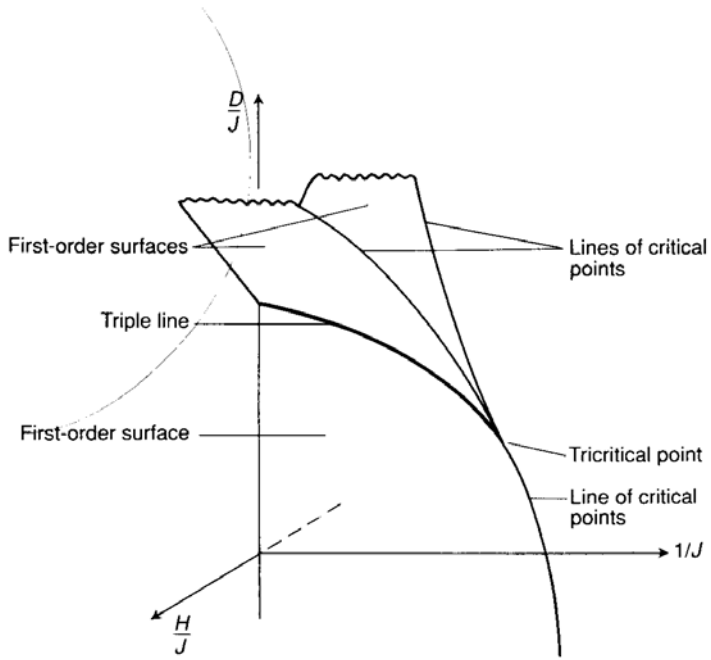
$$t_i = (1 - s_i)/2.$$

(b)



Modelo de Ising con S=1.

$$\mathcal{H} = -J\sum_{\langle ij\rangle} s_i s_j - K\sum_{\langle ij\rangle} s_i^2 s_j^2 - D\sum_i s_i^2$$
$$- L\sum_{\langle ij\rangle}(s_i^2 s_j + s_i s_j^2) - H\sum_i s_i, \qquad s_i = \pm 1, 0.$$



Potts Model:

$$\mathcal{H} = -J \sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j}.$$

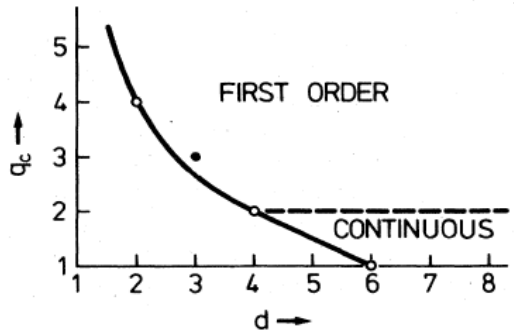


FIG. 2. Schematic plot of $q_c(d)$, the critical value of q beyond which the transition is mean-field—like (first order for $q > 2$ and continuous for $q \leq 2$). The known points $q_c(2)=4$, $q_c(4)=2$, and $q_c(6)=1$ are denoted by open circles. The black circle indicates the assumed first-order transition for $d=3$, $q=3$.

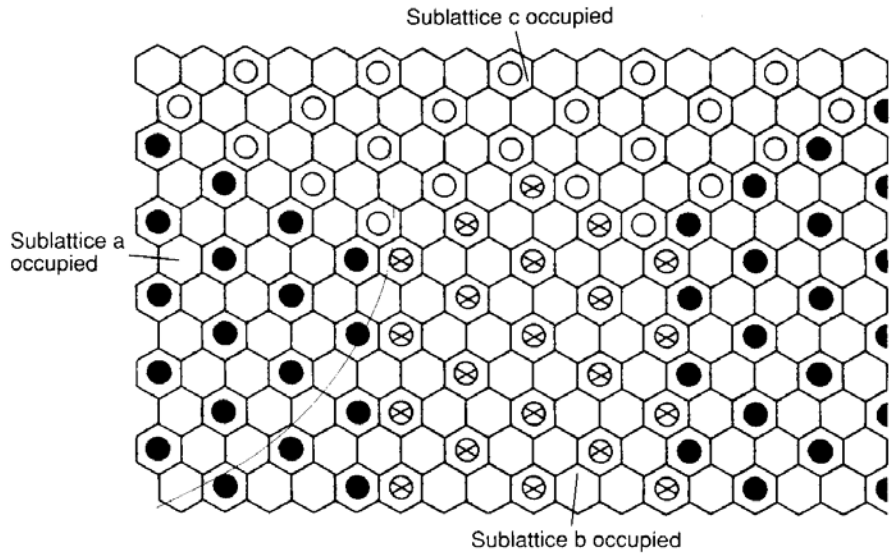


Fig. 3.6. Krypton adsorbed on the basal plane of graphite showing coexisting regions of the three ground states. After Kardar, M. and Berker, A. N. (1982). *Physical Review Letters*, **48**, 1552.

Modelos XY y Heisenberg.

$$\mathcal{H} = -J_z \sum_{\langle ij \rangle} s_i^z s_j^z - J_\perp \sum_{\langle ij \rangle} (s_i^x s_j^x + s_i^y s_j^y) - H \sum_i s_i^z$$

$$\mathcal{H} = -J \sum_{\langle ij \rangle} \vec{s}_i . \vec{s}_j - H \sum_i s_i^z .$$

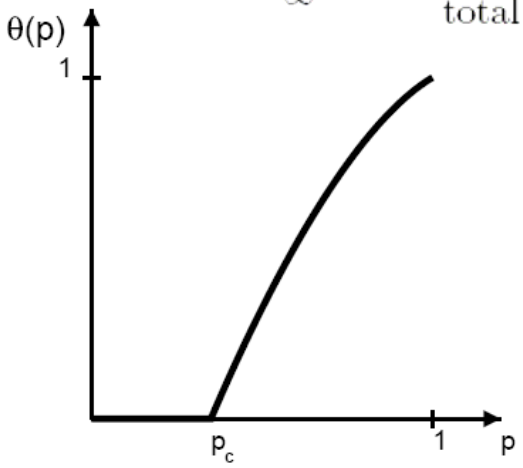
Discutir de nuevo Universalidad y "crossover".

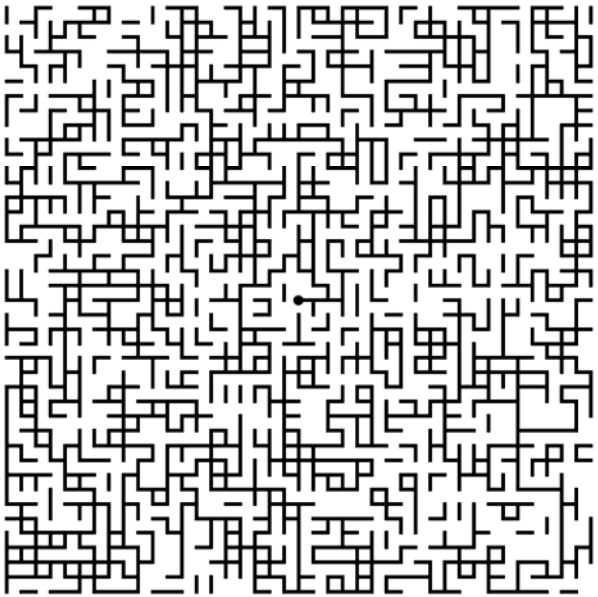
Table 3.1. Universality classes

Universality class	Symmetry of order parameter	α	β	γ	δ	ν	η	Physical examples
2-d Ising	2-component scalar	0 (log)	1/8	7/4	15	1	1/4	some adsorbed mono e.g. H on Fe
3-d Ising	2-component scalar	0.10	0.33	1.24	4.8	0.63	0.04	phase separation, flu order-disorder e.g. β
3-d X-Y	2-dimensional vector	0.01	0.34	1.30	4.8	0.66	0.04	superfluids, supercon
3-d Heisenberg	3-dimensional vector	−0.12	0.36	1.39	4.8	0.71	0.04	isotropic magnets
mean-field		0 (dis.)	1/2	1	3	1/2	0	
2-d Potts, q=3	q-component scalar	1/3	1/9	13/9	14	5/6	4/15	some adsorbed mono
q=4		2/3	1/12	7/6	15	2/3	1/4	e.g. Kr on graphite

Un último modelo: Percolación.

$$P_\infty = \frac{\text{number of sites in the spanning cluster}}{\text{total number of occupied sites}}.$$





(a)

(b)

(c)

FIG. 3. Site percolation clusters on a 2D square lattice at $p = 0.45$ (a), $p = 0.5927 = p_c$ (b), and $p = 0.7$ (c).

$$P_\infty \sim (p - p_c)^\beta,$$

$$n_s(p) = \frac{\text{mean number of clusters of size s}}{\text{total number of lattice sites}}.$$

lattice	d	q	p_c (site)	p_c (bond)
linear chain	1	2	1	1
square	2	4	0.59275	$\frac{1}{2}$
triangular	2	6	$\frac{1}{2}$	0.3473
honeycomb	2	3	0.698(3)	0.6527
diamond	3	4	0.4299(8)	0.3886(5)
simple cubic	3	6	0.3117(3)	0.2492(2)
bcc	3	8	0.24674(7)	0.1795(3)
fcc	3	12	0.1998	0.1198(3)

$$w_s = \frac{s n_s}{\sum_s s n_s},$$

$$R_s{}^2 = \frac{1}{s}\sum_{i=1}^{s}(\mathbf{r}_i - \overline{\mathbf{r}})^2,$$

$$S(p) = \sum_s s w_s = \frac{\sum_s s^2 n_s}{\sum_s s n_s}.$$

$$\xi^2 = \frac{\sum_s s^2 n_s \, R_s{}^2}{\sum_s s^2 n_s}.$$

quantity	functional form	exponent	$d = 2$	$d = 3$
order parameter	$P_\infty \sim (p - p_c)^\beta$	β	5/36	0.403(8)
mean size of finite clusters	$S(p) \sim \lvert p - p_c\rvert^{-\gamma}$	γ	43/18	1.73(3)
connectedness length	$\xi(p) = \lvert p - p_c\rvert^{-\nu}$	ν	4/3	0.89(1)
cluster distribution	$n_s \sim s^{-\tau} \quad (p = p_c)$	τ	187/91	2.2