

Transiciones de Fase (Fenómenos Críticos) en Materia Condensada.

¿Por qué estudiar transiciones de fase?

- Porque es un fenómeno físico de la naturaleza (y útil).
- Porque es difícil. (teórica y experimentalmente).
- Porque ilustra muchas técnicas y conceptos importantes en física.... (universalidad, scaling, simetría, invarianza de escala ...)
- Porque es un ejemplo (o es "el ejemplo") de estudio científico de Sistemas Complejos. (Propiedades emergentes)

¿por qué en Materia Condensada? ... Por que no?

Hay transiciones de fase en todas partes.

Historia :

- 1873: Ecuación de estado de van der Waals.
- 1908: Licuación del Helio. Transición superconductor.
- 1905-1928: Mecánica Cuántica. Fluidos Cuánticos ideales. Transición de Bose-Einstein. Transición de superfluidez del Helio4 (1937).
- 1925 Modelo de Ising.
- 1934 Solución de Bragg y Williams. (Campo Medio).
- 1935. Lev D. Landau. Teoría General de Campo Medio (Mean Field).
- 1944 Solución de Onsager del Modelo de Ising 2d.
- 1954-57: Simulaciones Numéricas.
- 1965 -1966: Widom- Kadanoff. Teoría moderna de TF.
- 1971: K.G. Wilson. Grupo de Renormalización.

La transición de Fase más Conocida.

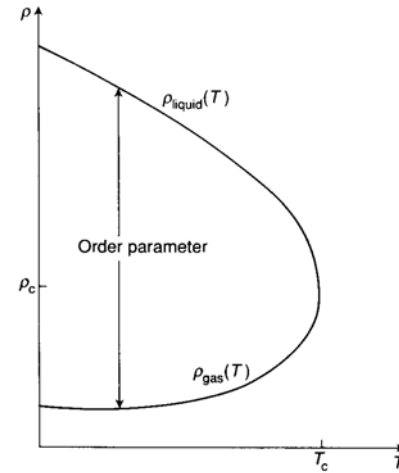
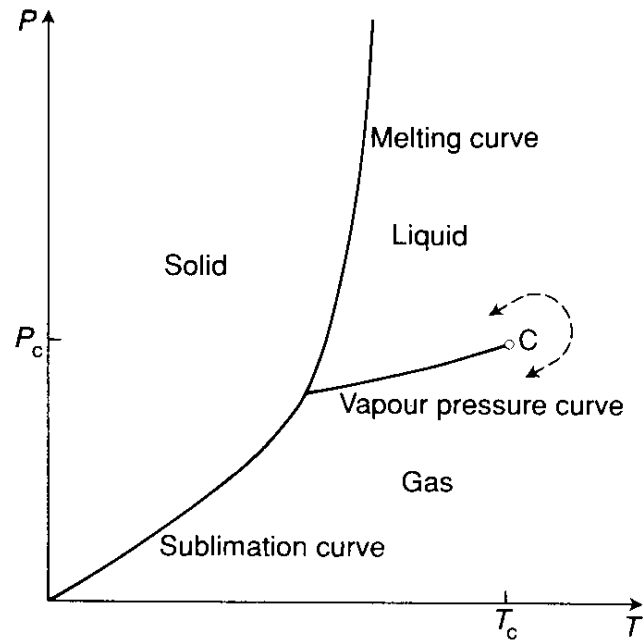


Fig. 1.2. Values of the densities of the coexisting liquid and gas along the vapour pressure curve. $(\rho_{\text{liquid}}(T) - \rho_{\text{gas}}(T))$ is the order parameter for the liquid-gas transition.

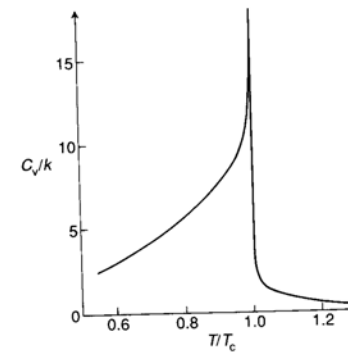
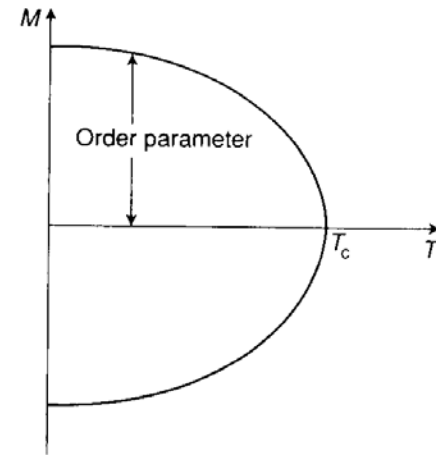
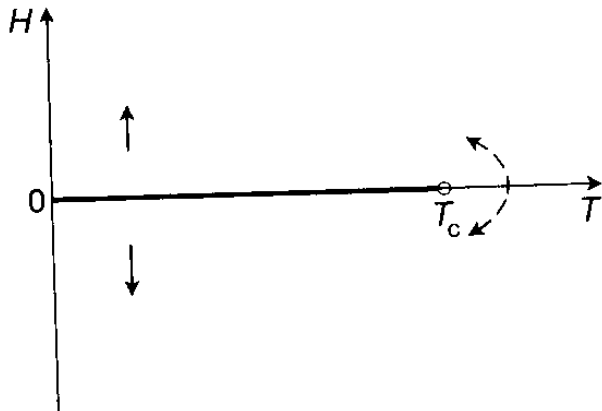
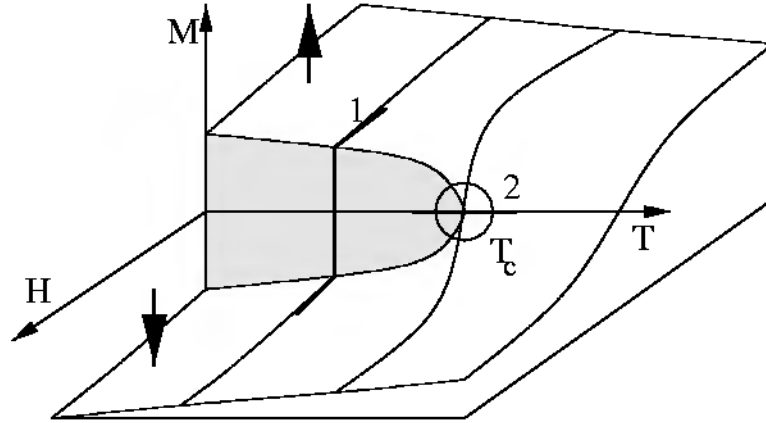


Fig. 1.3. Specific heat at constant volume of argon measured on the critical isochore, $\rho = \rho_c$. After Fisher, M.E. (1964). *Physical Review*, **136A**, 1599.

La transición de Fase más sencilla (de estudiar).



Otras transiciones de fase.

Table 1.1. Examples of the diversity of phase transitions found in nature

Transition	Example	Order parameter
ferromagnetic ^a	Fe	magnetization
antiferromagnetic ^a	MnO	sublattice magnetization
ferrimagnetic ^a	Fe ₃ O ₄	sublattice magnetization
structural ^b	SrTiO ₃	atomic displacements
ferroelectric ^b	BaTiO ₃	electric polarization
order-disorder ^c	CuZn	sublattice atomic concentration
phase separation ^d	CCl ₄ +C ₇ F ₁₆	concentration difference
superfluid ^e	liquid ⁴ He	condensate wavefunction
superconducting ^f	Al, Nb ₃ Sn	ground state wavefunction
liquid crystalline ^g	rod molecules	various

Situaciones más complicadas.

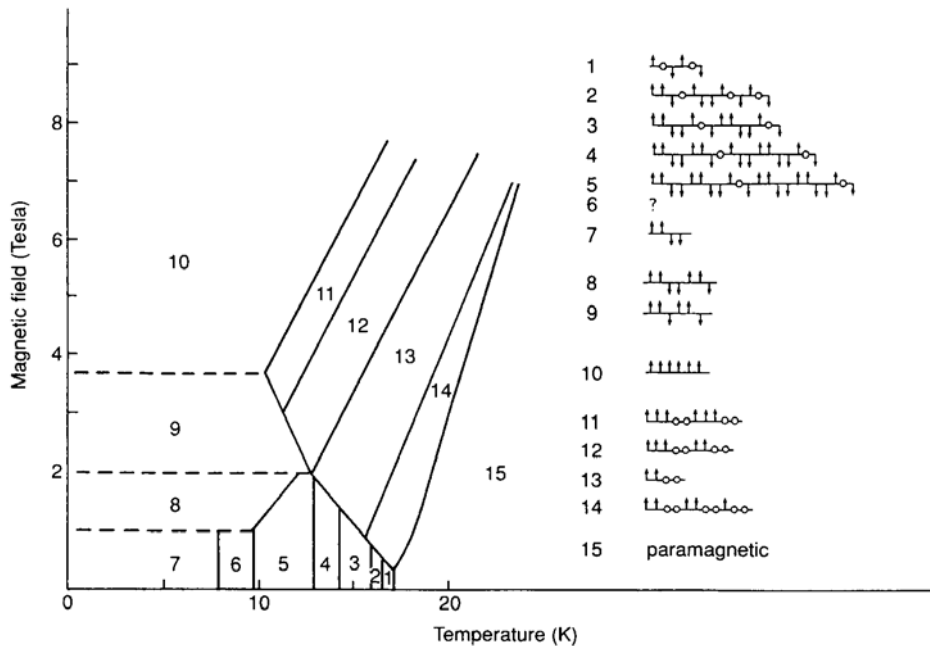


Fig. 1.6. The ferrimagnetic phases of cerium antimonide. The relative ordering of successive ferromagnetic planes in each phase is indicated in the Figure. \circ denotes a plane with a net magnetization of zero. After Rossat-Mignod, J., Burlet, P., Bartholin, H., Vogt, O., and Lagnier, R. (1980). *Journal of Physics C: Solid State Physics*, **13**, 6381, Institute of Physics Publishing Limited.

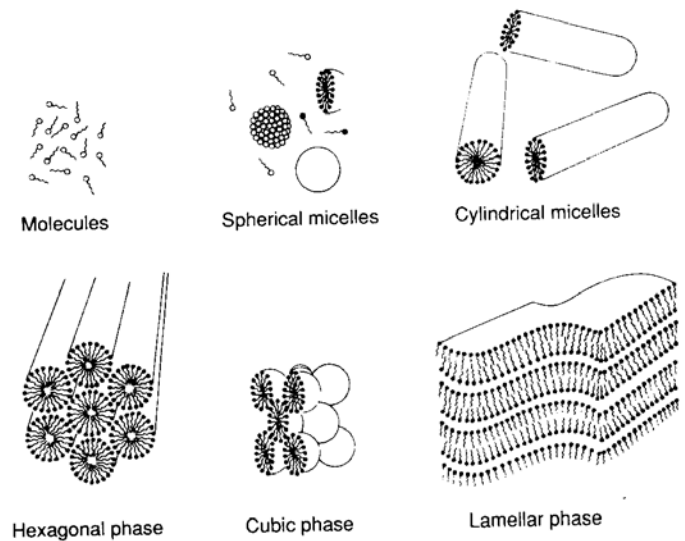


Fig. 1.7. Schematic drawings of the idealised structures of surfactant molecules that can form in solution as the surfactant concentration is increased. After Corkhill, J. M. and Goodman, J. F. (1969). *Advances in Colloid and Interface Science*, **2**, 297.

Estructura del estado crítico: efecto de un cambio de escala

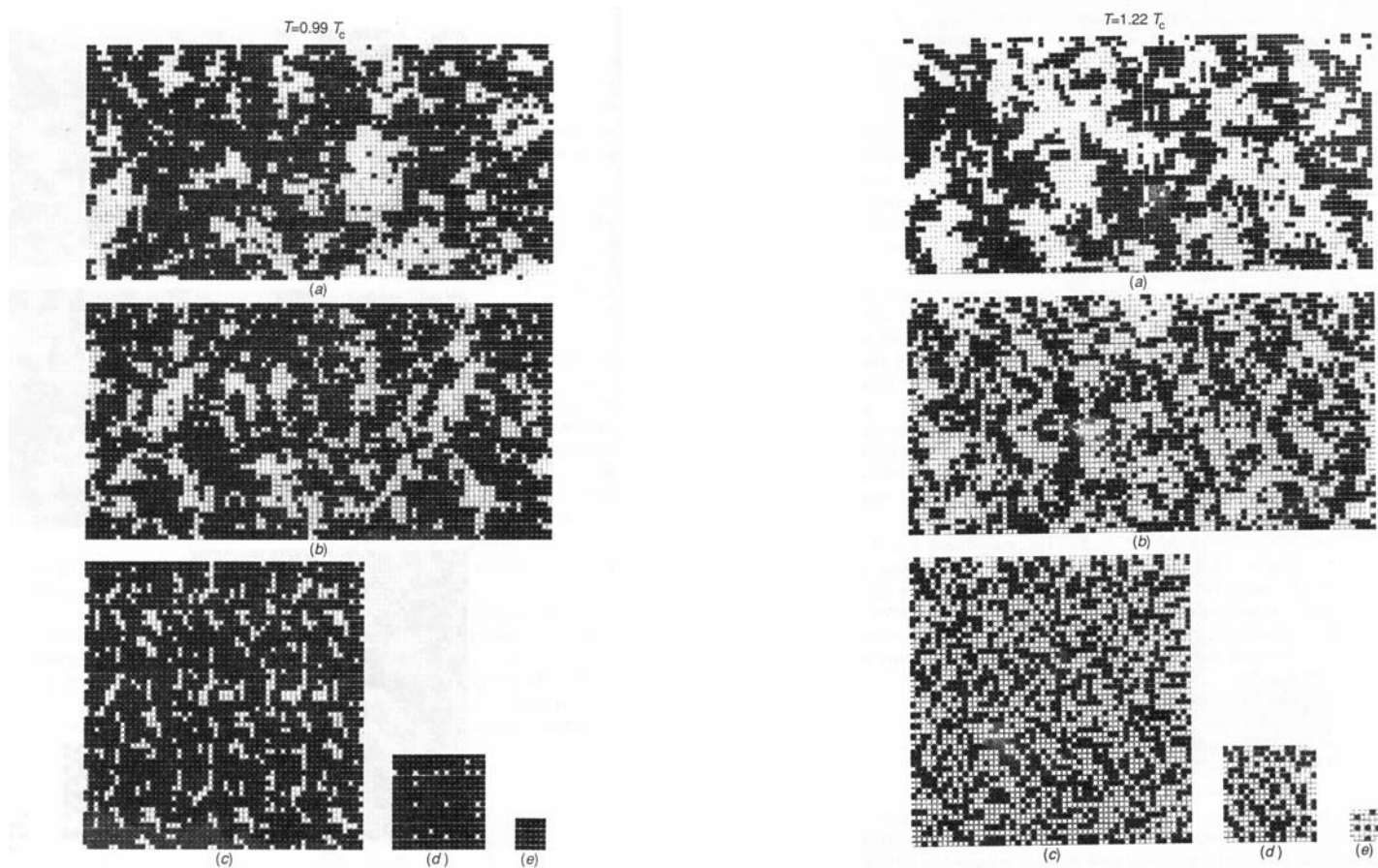


Fig. 1.10. As Fig. 1.8 but with a starting temperature $T = 0.99T_c$. Fluctuations relative to the ordered state are suppressed by the change in length scale and the system flows towards zero temperature. After Wilson, K. G. (1979). *Scientific American*, **241**, 140.

El punto crítico es invariante
bajo un cambio de escala !

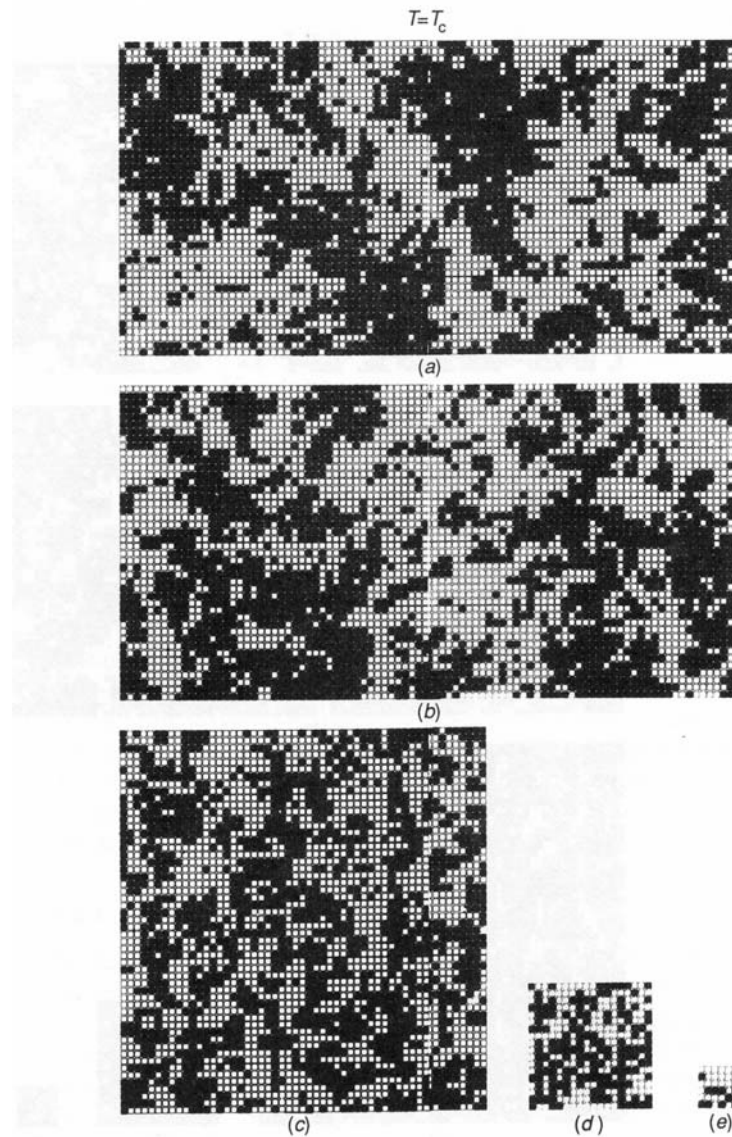


Fig. 1.9. As Fig. 1.8 but with a starting temperature $T = T_c$. Because the correlation length is initially infinite there is no change in the ordered state under iteration of the renormalization group and the system remains at the critical temperature. After Wilson, K. G. (1979). *Scientific American*, **241**, 140.

Table 2.1. The relation of the thermodynamic variables pertinent to a magnetic system to the partition function

Thermodynamic variables for a magnet

First law: $dU = TdS - MdH$

Partition function

$$Z(T, H) = \sum_r e^{-\beta E_r}$$

↓

Free energy

$$\mathcal{F} = -kT \ln Z$$

↙

Internal energy

$$U = -\frac{\partial \ln Z}{\partial \beta}$$

↓

Specific heat (constant H)

$$C_H = \left(\frac{\partial U}{\partial T}\right)_H$$

↓

Entropy

$$S = -\left(\frac{\partial \mathcal{F}}{\partial T}\right)_H = (U - \mathcal{F})/T$$

↓

Specific heat (constant X = H, M)

$$C_X = T \left(\frac{\partial S}{\partial T}\right)_X$$

↘

Magnetization

$$M = -\left(\frac{\partial \mathcal{F}}{\partial H}\right)_T$$

↓

Isothermal susceptibility

$$\chi_T = \left(\frac{\partial M}{\partial H}\right)_T$$

Table 2.2. The relation of the thermodynamic variables pertinent to a fluid system to the partition function

Thermodynamic variables for a fluid

First law: $dU = TdS - PdV$

Partition function

$$Z(T, V) = \sum_r e^{-\beta E_r}$$

↓

Free energy

$$\mathcal{F} = -kT \ln Z$$

↙

Internal energy

$$U = -\frac{\partial \ln Z}{\partial \beta}$$

↓

Specific heat (constant V)

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V$$

↓

Entropy

$$S = -\left(\frac{\partial \mathcal{F}}{\partial T}\right)_V = (U - \mathcal{F})/T$$

↓

Specific heat (constant X = V, P)

$$C_X = T \left(\frac{\partial S}{\partial T}\right)_X$$

↘

Pressure

$$P = -\left(\frac{\partial \mathcal{F}}{\partial V}\right)_T$$

↓

Isothermal compressibility

$$\kappa_T = -\frac{1}{V} \left(\frac{\partial V}{\partial P}\right)_T$$

Clasificación de las transiciones de fase:

-Discontinuas o 1° Orden: discontinuidad finita en las primera derivadas de la energía libre. Si la entropía es discontinua tenemos asociado un calor latente de la transición $T_c\Delta S$.

- Continuas o de 2° Orden: La primera derivada es continua pero las segundas son discontinuas. Punto crítico.

Están caracterizadas por exponentes críticos.

Función y longitud de correlación.

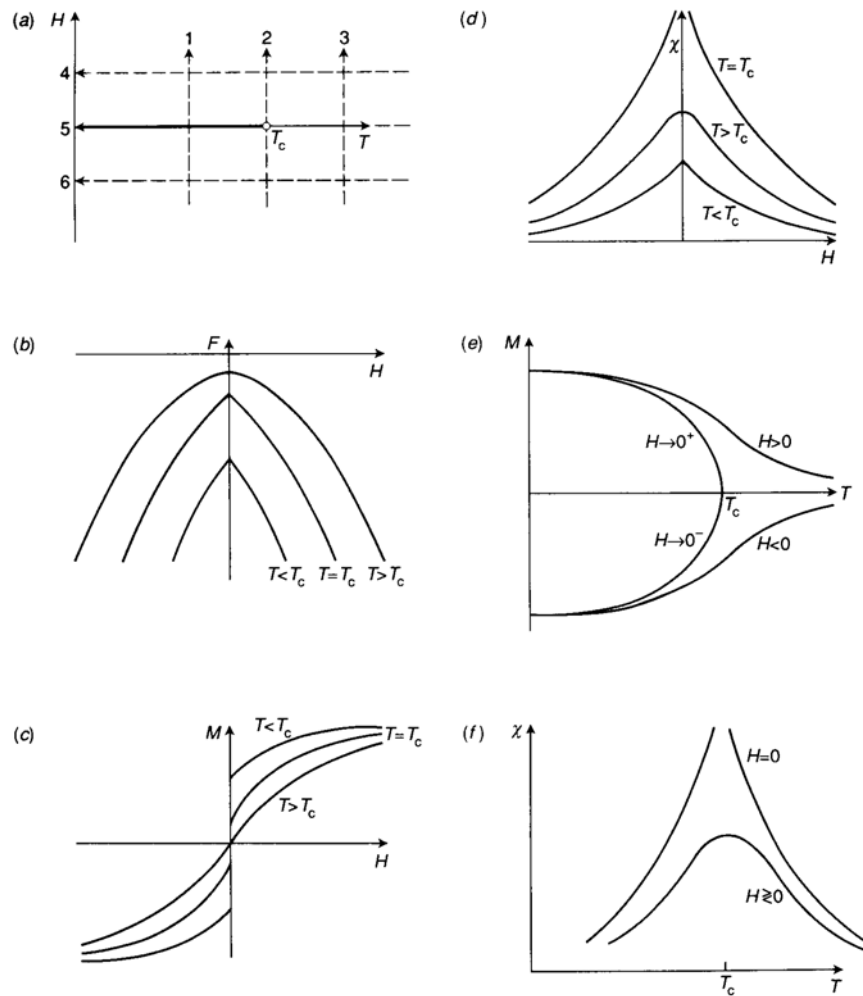
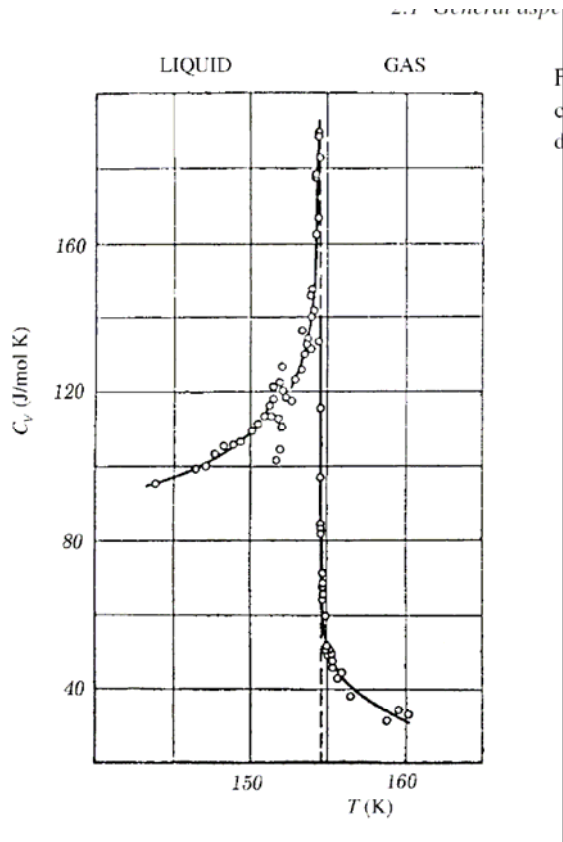


Fig. 2.1. (a) Phase diagram of a simple ferromagnet. There is a line of first-order transitions along $H = 0$ which ends at a critical point at $T = T_c$. (b) Field dependence of the free energy. (c) Field dependence of the magnetization. (d) Field dependence of the susceptibility. (e) Temperature dependence of the magnetization. (f) Temperature dependence of the susceptibility.

Exponentes críticos:

$$t = (T - T_c)/T_c$$



$$F(t) \sim |t|^\lambda . \quad \lambda = \lim_{t \rightarrow 0} \frac{\ln |F(t)|}{\ln |t|}$$

Table 2.4. Definitions of the most commonly used critical exponents for a fluid system

Specific heat at constant volume V_c	$C_V \sim t ^{-\alpha}$
Liquid-gas density difference	$(\rho_l - \rho_g) \sim (-t)^\beta$
Isothermal compressibility	$\kappa_T \sim t ^{-\gamma}$
Critical isotherm ($t = 0$)	$P - P_c \sim \rho_l - \rho_g ^\delta \operatorname{sgn}(\rho_l - \rho_g)$
Correlation length	$\xi \sim t ^{-\nu}$
Pair correlation function at T_c	$G(\vec{r}) \sim 1/r^{d-2+\eta}$

Table 2.3. Definitions of the most commonly used critical exponents for a magnetic system

Zero-field specific heat	$C_H \sim t ^{-\alpha}$
Zero-field magnetization	$M \sim (-t)^\beta$
Zero-field isothermal susceptibility	$\chi_T \sim t ^{-\gamma}$
Critical isotherm ($t = 0$)	$H \sim M ^\delta \operatorname{sgn}(M)$
Correlation length	$\xi \sim t ^{-\nu}$
Pair correlation function at T_c	$G(\vec{r}) \sim 1/r^{d-2+\eta}$

Universalidad

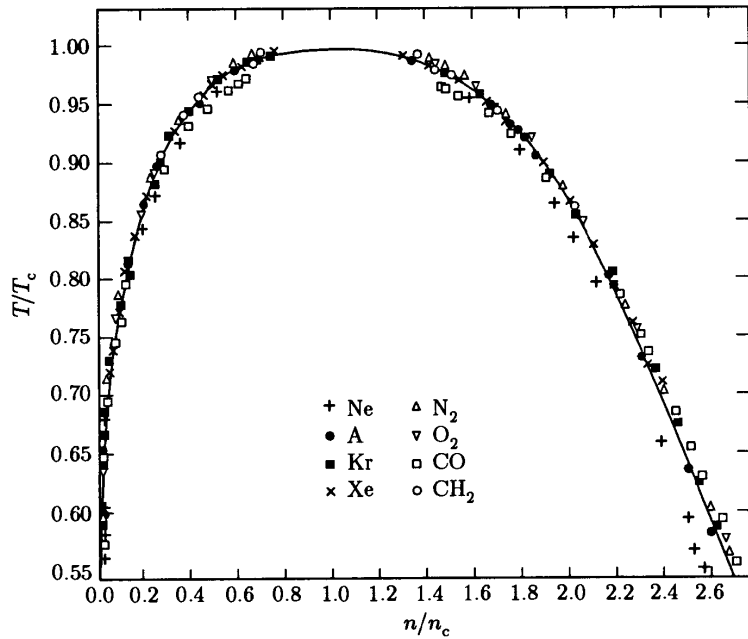


Fig. 4.4.4. Phase boundary in units of reduced temperature and density for eight different molecular fluids near their liquid-gas transitions. Note the universal behavior and the fact that the solid line is $\Delta\phi \propto (T_c - T)^\beta$ with $\beta = 1/3$ rather than the mean-field result $\beta = 1/2$. [E.A. Guggenheim, *J. Chem. Phys.* **13**, 253 (1945).]

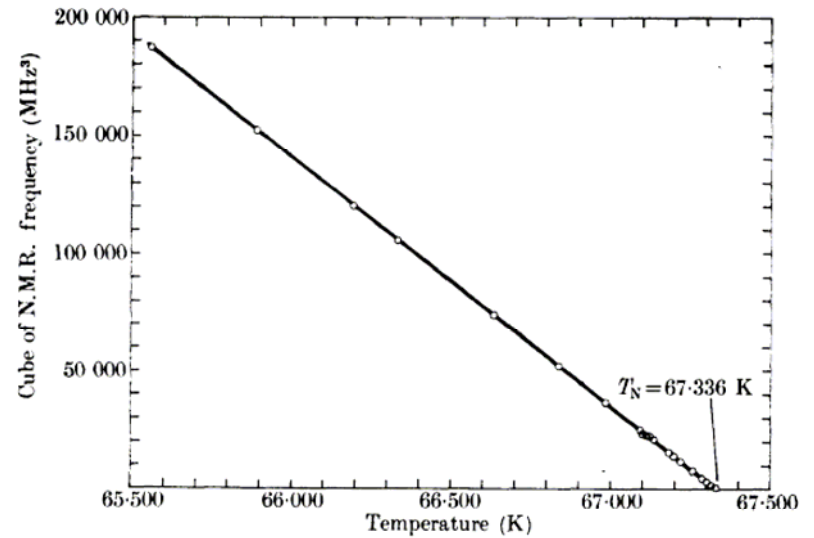


Fig. 1.9. Dependence upon temperature of the cube of the zero-field magnetization for MnF_2 . Since MnF_2 is an antiferromagnet instead of a ferromagnet, the critical temperature is denoted by T_N rather than by T_c . After Heller and Benedek (1962).

Relaciones entre exponentes críticos:

$$\chi_T(C_H - C_M) = T \left(\frac{\partial M}{\partial T} \right)_H^2. \quad \alpha + 2\beta + \gamma \geq 2.$$

Relaciones de scaling:

Fisher	$\gamma = \nu(2 - \eta)$
Rushbrooke	$\alpha + 2\beta + \gamma = 2$
Widom	$\gamma = \beta(d - 1)$
Josephson	$\nu d = 2 - \alpha$