are very dissimilar: the similarity of their thermodynamic consequences suggests that any potential intermediate between these two extremes will lead again to results the same as those shared by these two models.

A final point is the following: In the above problem, namely a one-dimensional Ising chain with *nm* sites and interactions between nearest neighbours and between spins separated by exactly *n* sites, how must we make the passage to the thermodynamic limit in order to get singular transition points? The question may be rigorously answered: as long as n and m both $\rightarrow \infty$ (even though $n \rightarrow \log R$ and $m \rightarrow R$ for example) then a true phase transition ensues. However it is worth noting that even the simplest departure from the conventional nearest-neighbour one-dimensional Ising model, namely the $n \times 2$ lattice with nearest-neighbour interactions, already has significant differences from the $n \times 1$ nearest neighbours case. Although of course it cannot exhibit any singular thermodynamic behaviour, this $n \times 2$ model * does have a specific heat, C_V, which as $T \rightarrow 0$ is proportional to $\exp(-4J/kT)$; this is to be compared with $\exp(-4J/kT)$ for the usual two-dimensional Ising model and $\exp(-2J/kT)$ for the usual one-dimensional one.

This is because the $n \times 2$ already shares with the $n \times m$ case the feature that to disturb the order at low temperatures it is necessary to disturb order in two directions from a given lattice site - this involves an energy gap of $2 \times (2J)$: in contrast, in the $n \times 1$ case only one direction is involved and the relevant gap is $2J$.

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* The solution of this $n \times 2$ problem is analogous to the simple one-dimensional nearest-neighbours Ising problem, except that the unit cell now has a 4×4 matrix instead of a 2×2. Calculation of the largest eigenvalue and consequent thermodynamics is tedious but straightforward, and yields the above result as $T \rightarrow 0$.

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ON A GENERALISED APPROACH TO FIRST AND SECOND ORDER MAGNETIC TRANSITIONS

B. K. BANERJEE
Department of Physics, The University, Newcastle upon Tyne

Landau [1] and Lifshitz [2] developed the thermodynamic theory of second order phase transitions. They showed that near the transition point, where the order parameter (J) can assume infinitely small values, the thermodynamic potential Φ can be expanded in a Taylor series. Thus

$$\Phi(T, P, J) = \Phi_0 + aJ^2 + bJ^4 + \dots , \qquad (1)$$

where the coefficients a and b are functions of the pressure (P) and the temperature (T), and Φ_0 is a constant. An equilibrium state is observed for the system when $\partial\Phi/\partial J = 0$. Vonsovskii [3] and Ginzburg [4] have successfully applied the above theory for the Curie point transitions in ferro-

magnets, where $J = J_T/J_0$. J_T and J_0 are spontaneous magnetisations per unit mass at temperatures T^0K and 0^0K respectively. In Ginzburg's formulation the magnetostatic field energy is also included, yielding

$$\Phi = \Phi_0 + aJ^2 + bJ^4 - JH \qquad (2)$$

limiting the expansion to J^4. At equilibrium near the Curie point, the above reduces to

$$\alpha\sigma + \beta\sigma^3 = H , \qquad (3)$$

where α, β = constants incorporating a, b and J_0 and σ = experimentally observed specific magnetisation.

A thermodynamic analysis of the properties of a second order phase transition that when the coefficient of $\sigma^2 = 0$, is most always positive. Experimental values for A are obtained as the slopes of the linear isotherm plots $\Delta H/\sigma$ against σ^2, as will be apparent from eq. (3). The Curie point is given by that value of T for which $a = 0$. Details of the above arguments will be found in English language as given by Below [5].

Bean and Rodbell [6] have developed a theory of first order magnetic transitions and applied it successfully to the case of MnAs. As they have shown, the first order transition in a compressible material is a direct consequence of incorporating in the molecular field model an exchange interaction that is strongly dependent upon interatomic spacing. Their eq. 5 which expresses the Gibbs free energy minimum can be written as follows:

$$\left(\frac{\partial G}{\partial \sigma} \right)_{p,\,th} = -A_1 + A\sigma^2 + B\sigma^4 + \ldots \;, \quad (4)$$

The coefficients A_1, A and B depend on pressure, temperature, compressibility, dependence of the exchange constant on interatomic spacing etc. σ is the ratio of magnetisation per unit volume at the temperature of observation and that at $0^\circ K$. G = Gibbs free energy per unit volume, N = number of particles per unit volume, k = Boltzmann's constant and T_0 - the transition temperature if the material was incompressible. For small values of σ, Bean and Rodbell find that A should be equal to zero at the Curie point. They also point out that for the transition to be of the second order, in addition to A being zero, B should stay positive, otherwise a minimum energy is reached for $\sigma \neq 0$, and this results in a first order transition.

It seems to us that this essential similarity between the Landau-Lifshitz and the Bean-Rodbell approach has not been detected or emphasised previously. Obviously, both the eqs. (3) and (4) can be incorporated into one, with the proviso that when the coefficient of $\sigma^2 = 0$, the nature of the transition can be determined from the experimentally observed sign of the coefficient of σ^4. It has to be admitted here that, limiting himself to second order transitions, Belov [7] recognised that the Langevin function can be expanded into a power series of σ. But he also showed the quantitative disagreement between such an approach and the experimentally observed data

for nickel. With this disagreement in mind, we can expect that, though the Bean-Rodbell approach for a first order transition is qualitatively similar to a generalised thermodynamic one, there may be disagreement between the experimentally observed values of A and B and the predicted values from eq. (4). This, however, is irrelevant so far as the distinction between the two signs of B is concerned.

In order to check the idea that first and second order transitions can be distinguished from the sign of the slope of isotherm plots of H/σ against σ^2, we have extracted the relevant data from fig. 11 of Bean and Rodbell [6] for MnAs. Least square fits for straight line solutions were obtained and the values of the slopes obtained are presented below (table 1). Only those values of the magnetic field H were used for which the material was still in a ferromagnetic phase.

Table 1

Temperature ($^\circ$C)	Field range (kOe)	Slope (H/σ versus σ^{-2})
34 ± 2	15–40	-24.16×10^{-6}
53 ± 2	15–60	-35.02×10^{-5}
64.5 ± 2	15–60	-7.95×10^{-6}

If the Bean-Rodbell expression for the coefficient B (eq. 6 of their paper) was quantitatively applicable, one would expect the absolute value of the slope to decrease continuously with temperature. The observed departure may be due to the essential inaccuracy of the method of replotting, a variation of η (see eq. (4) of Bean and Rodbell) with temperature and other reasons. But from the negative sign of all the values for slope in table 1, we suggest that this method of plotting the observed results could provide us with a tool to distinguish first order magnetic transitions from second order ones by purely magnetic methods.

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